Kunming School 2018: Lab assignments day 6

Rony Keppens & Jannis Teunissen

1 The two-stream instability

One of the things you cannot capture with a fluid code are so-called *wave-particle interactions*. Examples of such wave-particle interactions are Landau damping and the two-stream instability. Today, we will use a simple 1D PIC (particle-in-cell) code to investigate the two-stream instability.

We will use two beams of electrons, with mean velocities v_1 and v_2. Both beams also have a thermal velocity of v_{th}. These beams are neutralized by a static background of positive ions, so that we obtain a neutral plasma. Important plasma parameters are the plasma frequency

$$w_p = \sqrt{\frac{n_e e^2}{m_e \varepsilon_0}}$$

and the Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}},$$

where n_e is the electron density, e the electron charge, m_e the electron mass, ε_0 the permittivity of vacuum, k_B Boltzmann's constant and T_e the electron temperature. In 1D, $k_B T_e$ is given by $m_e v_{\mathrm{th}}^2$.

- After you have downloaded the code, try to find the essential PIC components: the particle mover, the mapping of particles to grid densities, and the computation of the field on the grid.
- Compile the program with `gfortran two_stream.f90 -o two_stream -Wall`, and run the simulation like this: `time ./two_stream`. How long does it take?
- Now compile the program with `gfortran two_stream.f90 -o two_stream -Wall -O2`, and run it again with `time ./two_stream`. How long does it take now?
- Generate a movie of the simulation results by running the included script `bash gen_movie.sh`. How long does it take for the system to become unstable?
- You can change the velocity of the beams. Do a run in which v_1 and v_2 are equal. Is there an instability now?

2 A rotating particle

We will study the motion of a rotating particle in a two-dimensional (x, y) coordinate system. The motion is described by the following equations

$$\begin{aligned}\partial_t \vec{x}(t) &= \vec{v}(t) \\ \partial_t \vec{v}(t) &= -\hat{x}(t)/x(t)^2,\end{aligned}$$

and the initial position and velocity are $\vec{x}_0 = (1, 0)$ and $\vec{v}_0 = (0, 1)$. The analytic solution to this problem is

$$\begin{aligned}\vec{x}(t) &= [\cos(t),\, \sin(t)] \\ \vec{v}(t) &= [-\sin(t),\, \cos(t)].\end{aligned}$$

1. Solve the equations of motion up to $t = 6\pi$ using the forward Euler method:

$$\begin{aligned}\vec{x}_{t+1} &= \vec{x}_t + \vec{v}_t \Delta t \\ \vec{v}_{t+1} &= \vec{v}_t + \vec{a}_t \Delta t.\end{aligned}$$

   Use a time step of $h = \pi/10$. What does the solution look like? Is the system's energy $(\frac{1}{2}v^2 + 1/|\vec{x}|)$ conserved?

2. Now use a higher order method such as the midpoint method:

$$\begin{aligned}\vec{x}_{t+1/2} &= \vec{x}_t + \vec{v}_t \Delta t/2 \\ \vec{v}_{t+1/2} &= \vec{v}_t + \vec{a}_t \Delta t/2 \\ \vec{x}_{t+1} &= \vec{x}_t + \vec{v}_{t+1/2} \Delta t \\ \vec{v}_{t+1} &= \vec{v}_t + \vec{a}_{t+1/2} \Delta t.\end{aligned}$$

   Compare the predicted trajectory with the forward Euler solution. Is energy conserved? Do you think a higher order Runge-Kutta method would be suitable for long-running simulations of this system?

3. Finally, use a *leap-frog* scheme to advance the particle in time:

$$\begin{aligned}\vec{x}_{t+1/2} &= \vec{x}_t + \vec{v}_t \Delta t/2 \\ \vec{v}_{t+1} &= \vec{v}_t + \vec{a}_{t+1/2} \Delta t \\ \vec{x}_{t+1} &= \vec{x}_{t+1/2} + \vec{v}_{t+1} \Delta t/2.\end{aligned}$$

   Does this scheme conserve energy?